The form factor f_0 for a stationary atom is approximated by a sum of the Gaussian functions (Forsyth & Wells, 1955)

$$f_{i0} = A_{i1} \exp(-a_{i1}s^2) + A_{i2} \exp(-a_{i2}s^2) + A_{i3} \exp(-a_{i3}s^2)$$
(2)

with $a_{i3}=0$, where $s=\sin \theta/\lambda$. Equation (1) is, then, easily integrated, and we obtain

$$\begin{array}{c}
\varrho_{i}(0) = 8\pi^{3/2} \sum_{k=1}^{3} \frac{A_{ik}}{(a_{ik} + B_{i})^{3/2}} \\
P_{ij}(0) = 8\pi^{3/2} \sum_{k=1}^{3} \sum_{k'=1}^{3} \frac{G_{kk'}}{(g_{kk'})^{3/2}}
\end{array}$$
(3)

where

 $G_{kk}' = A_{ik}A_{jk}', g_{kk}' = a_{ik} + B_i + a_{jk}' + B_j$

and B_i is the isotropic temperature factor of the *i*th atom. Although the relations (3) are not suitable at very low temperatures, they give sufficiently good approximations for most purposes. Parameters A_k and a_k in equation (2) were obtained by Moore (1963), and also by Hosoya & Satake (1965). Hosoya & Satake's parameters for the Mo range were used for calculation of the function $\rho(0)$ and the self Patterson function $P_{ii}(0)$ of various atoms. The results are shown in Table 1.

Practically, however, observed values are in general considerably smaller than these values because of termination. In order to calculate $\rho(0)$ and $P_{ii}(0)$ subjected to the termination effect, equations (1) were rewritten as

$$\varrho_{i}(0) = 32\pi \int_{0}^{1/\lambda} s^{2} f_{i} ds
P_{ii}(0) = 32\pi \int_{0}^{1/\lambda} s^{2} f_{i}^{2} ds$$
(4)

Substituting relation (2) into (4), values of $\rho_i(0)$ and $P_{i\ell}(0)$ were obtained numerically for various wavelengths. Typical results are shown in Fig.1. With Mo $K\alpha$, the termination is not very serious unless the temperature factor is too small, but with Cu $K\alpha$ the reduction of peak height is significant. Therefore, values for the Cu $K\alpha$ range are shown in Table 2. Tables 1 and 2 will be helpful for judging the atomic species, by reading through the column with a relevant *B* value.

The numerical calculations were carried out on OKITAC 5090H computer of this Institute. The author expresses his sincere thanks to Dr T. Ito of this Institute for his critical discussions.

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The crystal structure of CsUF₆*. By ABRAHAM ROSENZWEIG[†] and DON T. CROMER, University of California, Los Alamos Scientific Laboratory, Los Alamos, New Mexico 87544, U.S.A.

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CsUF₆ is rhombohedral, a=5.417 Å, $\alpha=95^{\circ}29.5'$, space group $R\overline{3}$, with one formula unit per unit cell. The U atom has 6F neighbors at 2.057 Å which make an octahedron slightly compressed along the threefold axis. The Cs atom has 12 neighbors, 6 at 3.101 and 6 at 3.147 Å. The compound is isostructural with KOsF₆.

The preparation of cesium uranium(V) hexafluoride was first reported by Penneman, Sturgeon & Asprey (1964), who found a rhombohedral lattice with hexagonal cell dimensions of $a = 8.036 \pm 0.003$, $c = 8.388 \pm 0.004$ Å. These dimensions suggest that the compound is isostructural with KOsF₆ whose structure, based on powder diffraction data, was reported by Hepworth, Jack & Westland (1956). Optical properties, cleavage and twinning were reported by Sturgeon, Penneman, Kruse & Asprey (1965). A partial absorption spectrum was reported by Asprey & Penneman (1964) and a detailed analysis of the absorption spectrum was made by Reisfeld & Crosby (1965), who assumed the centric space group $R\overline{3}$. CsPuF₆ (Penneman, Sturgeon, Asprey & Kruse, 1965) and CsNpF₆ (Asprey, Keenan, Penneman & Sturgeon, 1966) are isostructural with CsUF₆. The present structure determination was undertaken as a means of confirming the centric nature of the structure,

and of refining this structure type on the basis of singlecrystal data.

Crystals of CsUF₆ for this study were prepared by R.A. Penneman by recrystallization from cold, aqueous hydrogen fluoride solution. The crystals thus formed are pale greenish-blue rhombohedra up to 2 mm in diameter. Cleavage and twinning as reported by Sturgeon *et al.* (1965) were not observed on these crystals.

Precession photographs show the crystals to be rhombohedral with no systematic extinctions. The space group is thus R3, R3, R32, R3m, or R3m. Lattice constants were obtained from measurement of 2θ for Mo $K\alpha_1$ radiation $(\lambda = 0.70926 \text{ Å})$ for several high order reflections with a carefully aligned single-crystal orienter on a General Electric XRD-5 spectrogoniometer. The hexagonal cell constants are $a = 8.021 \pm 0.003$, $c = 8.430 \pm 0.003$ Å (the rhombohedral cell has a = 5.417 Å, $\alpha = 95^{\circ} 29^{\circ}.5$). These values differ significantly from those previously reported. The assumption of one formula unit per rhombohedral unit cell gives a calculated density of 5.145 g.cm⁻³. Intensities from an entire hemisphere were measured to $2\theta \simeq 50^{\circ}$ with Mo $K\alpha$ radiation with balanced Zr-Y filters and the fixed-counter,

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Table 1. Final least-squares parameters in CsUF₆ (rhombohedral cell)

	x	у	Ζ	$\beta_{11} \times 10^4$	$\beta_{22} \times 10^4$	$\beta_{33} imes 10^4$	$\beta_{12} imes 10^4$	$\beta_{13} imes 10^4$	$\beta_{23} \times 10^4$
U Cs	0 1	0 1	0 4	199 ± 3 270 + 4	β_{11} β_{11}	β_{11} β_{11}	121 <u>+</u> 4 134 + 7	β_{12} β_{12}	β_{12} β_{12}
F	$\overline{0.6789}$ ± 11	Õ·8036 ±11		333 ± 27	364 ± 27	412 ± 29	-15 ± 42	172 ± 43	243 ± 44
				q = 3	$79 \pm 0.08 \times 10^{-10}$)-6			

fixed-crystal technique. All reflections within this range were observed and intensities of symmetry-related reflections were averaged giving a total of 187 non-equivalent reflections. The crystal, bounded by seventeen planes, had a maximum diameter of 0.027 cm, and the necessary absorption corrections were applied by the Busing & Levy (1957) method using Burnham's (1962) program modified for the present geometry.

A three-dimensional Patterson map was computed. The map indicated that the heavy atoms are in the special posi-

Table 2. Observed and calculated structure factors for CsUF₆

Column headings are l, $10F_o/K$ and $10F_c^*/K$ (see text).

Нz	-2 K= -3	F= 1 K= 0	H=	2 K= 1	H =	3 K= 2	H =	4 K= 1	Hs	5 K. O
-4	85 77	-6 115 117	0	315 299	-4	135 135	- 2	136 141	-1	409 421
		-5 459 463	1	727 712	- 3	515 511	-1	493 516	0	120 125
H=	-1 K= -3	-4 218 223			- 2	176 177	0	129 142		
		-3 668 696	Ha	2 K= 2	-1	636 633	1	499 506	H =	5 K= 1
-5	83 74	-2 154 -94			0	79 +0				
-4	306 306	-1 920 924	- 3	217 210	1	514 510	H=	4 K= 2	-4	352 355
		0 375 377	-2	757 750	2	176 183			- 3	155 157
H=	-1 K= -2		-1	325 314			-5	139 135	-2	402 423
		F= 1 K= 1	0	766 769	н.	3 K = 3	-4	406 393	-1	131 134
-5	270 267		1	181 164			- 1	162 158	ō	414 440
-4	88 85	-1 307 300	;	A98 A70	- 5	1 10 1 17	- 5	515 551	ĩ	124 127
-3	551 577	0 340 921	•	0.0 0.9	- 4	612 661		165 166	•	120 127
-	• • • • • •	1 142 -98	64 w	3 83		140 141	÷	433 433		E
u-	0 43	1 142 10		2 44 - 2	- :	100 101		-23 -23	P .•	2 4- 2
	0 14 - 3					447 464	1	48 105		
- 6	383 370	PF 2 K3 - 3	-4	340 385	- 1	132 123	2	318 317	-3	333 344
-7	201 214	F 130 41.			0	430 435			- 2	144 149
	151 157	-> 338 314	н	3 K= -2	1	110 112	н÷	4 K. 3	-1	376 383
		-4 134 132			- 2	180 378			0	130 141
нэ	0 KH -2		- 5	382 377	3	76 80	-4	134 133	1	301 308
-	-	F≈ 2 Ks -2	-4	120 125			-3	405 401	2	79 76
-5	99 96		- 3	525 539	H =	4 K= -3	-2	172 171		
-4	500 480	-5 144 135					-1	419 412	He	5 K= 3
-3	224 221	-4 437 434	HI	3 K= -1	- 4	146 147	0	109 109		
		-3 72 60					1	335 349	- 2	293 300
н×	0 к≠ −1		-5	134 132	H=	4 K= -2	ž	96 104	-1	76 84
		F= 2 K= −1	-4	487 488		-	-		ō	263 264
-6	103 91		- 3	242 240	- 4	187 390	He	4 K= 4	ĩ	95 101
-5	387 375	-6 103 99	- 2	849 844	- 1	142 142			•	
-4	227 228	-5 416 418	-1	216 217		664 643	-1	00 04		4 4 2
-i	881 854	-5 173 179	- •	210 211	- 4			226 200		0 K2
- 2	205 204	-1 470 491		3 ** 0		/ ** 1		320 300		
•	207 204	-3 814 061	<i>n</i> •	3 4- 0	· ·	- v1	-1	125 124	-2	302 305
		-2 204 202					0	244 244		
нэ	1 K= -3	-1 934 928	-?	417 421	- 5	318 317	1	98 106	н.	6 K= -1
			-4	176 190	- 4	68 88				
-5	136 131	H= 2 K= 0	- 3	659 650	- 3	539 531	H=	5 X.= -2	- 2	123 126
-4	459 440		- 2	263 263	- 2	212 221			-1	369 377
		-6 312 316	-1	754 777	-1	616 628	-4	122 123		
H=	1 X* -2	-5 180 187	0	228 223			- 3	369 366	H A	6 ¥= 0
		-4 673 692			H =	4 K= 0	-2	85 88		
-5	379 349	-3 189 198	H¢	3 6 1 1					- 2	331 340
-4	149 139	-2 202 803			- 5	113 120	на	5 Ko -1	-1	111 114
-3	603 600	-1 252 247	- 5	148 150	- 2	434 430				112 121
-		0 821 839	-2	500 485		160 171	- 4	340 335		<i>J J L J L L</i>
Ma	1 Kr1			80 40	- 5					
•••-	• • • •	N- 7 K- 1		107 100		71 40		132 132	n.	0 K= 1
- 4	300 373	N- 1		370 370	-1	C 00	-2	-11 433		
- 6	211 211		-1	210 215	u.	200 282	-1	10- 145	-2	() 86
-?	40 HO	-0 121 118	0	141 740					-1	286 298
-4	600 592	-2 453 462	1	246 250	н=	4 K= 1	на	5 K# C	0	111 123
~3	210 205	-• 211 204								
-2	807 827	-3 706 689	HT	3 K= 2	- 5	403 386	-4	140 142		
		-2 297 279			- 4	146 142	-3	453 461		
		-1 1065 1071	-5	394 390	- 3	476 476	-2	131 144		

tions 0,0,0 and $\frac{1}{2}, \frac{1}{2}, \frac{1}{2}$ and that the fluorine atoms are in a set of general positions of space group $R\overline{3}$. A full-matrix least-squares refinement, including anisotropic thermal parameters, was made. The quantity minimized was $\Sigma w(F_o - F_c^*)^2$, where w is the weight based on counting statistics as given by Evans (1961), and

$$F_c^* = KF_c / \left\{ 1 + g \operatorname{Lp} \left[\frac{2(1 + \cos^4 2\theta)}{(1 + \cos^2 2\theta)^2} \right] F_c^2 \right\}^{\frac{1}{2}}$$

where K = scale factor, g = extinction parameter according to Zachariasen (1963), Lp = Lorentz and polarization factor and $F_c =$ the structure factor calculated in the ordinary way. The form factors for uranium and cesium were taken from Cromer & Waber (1965); that for fluorine was taken from *International Tables for X-ray Crystallography* (1962). Dispersion corrections (real part only) were applied to the uranium and cesium form factors (Cromer, 1965). The anisotropic temperature factors were in the form

$$\exp[-(h^2\beta_{11}+k^2\beta_{22}+l^2\beta_{33}+hk\beta_{12}+hl\beta_{13}+kl\beta_{23})].$$

The final parameters are given in Table 1. Table 2 lists the observed and calculated structure factors for which $R = \Sigma |\Delta F|/\Sigma |F_o| = 0.025$. Table 3 gives the magnitudes and orientations of the thermal ellipsoids.

Fig.1 is a stereo pair of the structure viewed approximately along the threefold symmetry axis. The uranium atom lies at a center of symmetry with six equidistant fluorine neighbors at 2.057 ± 0.006 Å. The fluorine atoms form an octahedral group slightly flattened along the threefold axis; the F–U–F angles are 91.9 and 88.1 \pm 0.2°. Each cesium atom has twelve fluorine neighbors: six at 3.147 \pm 0.006 Å, forming a trigonal antiprism, and six at 3.101 \pm 0.005 Å forming a puckered ring about the equatorial plane of the trigonal antiprism. The structure is indeed analogous to that of KOsF₆, and the centric interpretation of the absorption spectra (Reisfeld & Crosby, 1965) is confirmed.

On the basis of the paramagnetic resonance absorption of polycrystalline $CsUF_6$, Rigny & Plurien (1967) have



Fig. 1. Stereo view of C_sUF_6 approximately along the threefold axis. The letters X, Y, Z are at the ends of the vectors a, b, c so as to specify the location of the origin. The uranium positions are indicated by the letter U. Only the uranium at the point 1,0,1 has its thermal ellipsoid plotted. The cesium atom is at the center of the cell.

Table 3. Thermal ellipsoids in CsUF₆

		R.M.S.		Direction angles relative to the rhombohedral cell axes			
	Axis <i>i</i>	amplitude	B_i	α	β	γ	
Cs	$\left\{\begin{array}{c}1\\2\\3\end{array}\right\}$	0.182 ± 2 Å 0.220 ± 2	$2.61 \pm 5 \text{ Å}^2$ 3.84 ± 9	$ \left\{\begin{array}{c} 31\\ 90\\ 59 \end{array}\right. $	115 138 59	115 42 59	
U	$\left\{\begin{array}{c}1\\2\\3\end{array}\right\}$	0.150 ± 1	1.79 ± 3	$\begin{cases} 31\\ 90\\ 59 \end{cases}$	115 138 59	115 42 59	
F	1 2 3	0.198 ± 2 0.237 ± 8 0.188 ± 9 0.260 ± 9	3.03 ± 3 4.43 ± 31 2.79 ± 27 5.34 ± 36	37 ± 8 53 ± 8 92 ± 14	124 ± 12 53 ± 7 55 ± 12	70 ± 15 124 ± 6 41 ± 10	

found that the UF_6 octahedra are axially distorted. This observation is in agreement with the present results.

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Determination of particle size and strain in cold-worked magnesium by the method of variance. By N. K.

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The method of variance was used to determine particle size and strain in cold-worked magnesium. The results are discussed and compared with those obtained by line shape analysis.

X-ray line broadening investigation of deformed metals has mainly been confined to cubic structures. Few hexagonal metals and alloys have so far been studied. Lele & Anantharaman (1964) have studied the deformation of hexagonal magnesium by the method of integral width. They did not consider the individual contributions of particle size and strain to the line broadening but attributed the entire broadening to either size effects or strain effects. Recently Mitra & Misra (1967) have determined particle size and strain in cold-worked magnesium by line shape analysis using single-line technique. No other work on X-ray line broadening study of cold-worked magnesium has been reported. In this work we present the results of the application of the method of variance to the determination of particle size and strain in deformed magnesium.

The variance of a line profile is an explicit function of the range and hence depends largely on the error in the assumed background of the profile. The dependence of variance on the background error has been discussed by Berry (1966) and by Mitra & Misra (1966). Mitra & Misra (1966) have also developed a method of correcting the background error which has been applied in the present case. The variance of a line profile corrected for background varies linearly with range (in 2θ) in a region where the intensity decreases as the inverse square of the range. Considering the entire diffraction broadening to be due to particle size and strain, the variance of a line profile in 2θ is given by Wilson (1963); this may be rearranged according to Mitra (1964):

$$\frac{W\cos\theta}{\lambda\sigma} = \frac{1}{2\pi^2 p} + \frac{n^2\lambda}{\sigma\cos\theta} \cdot \frac{\langle e^2 \rangle}{d^2}$$
(1)

where p=t/K is the apparent particle size, t is the real particle size, d is the interplanar distance, $d/n = \lambda/2 \sin \theta$, θ is the Bragg angle, σ is the angular range in 2θ and $\langle e^2 \rangle$ is the variance of strain. The taper parameter in equation (1) has been assumed to be zero. By proper change of axes, reflexions of the type hkl, 2h2k2l etc. can be treated as $00l_0$, $002l_0$ etc. reflexions for each of which the Scherrer constant K=1 (Wilson, 1962). Since higher order reflexions are not available in the present case, we have used equation (1) to analyse each line separately, putting n=1. The line